An Information-Theoretic Neural Model Based on Concepts in Chinese Medicine

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***Abstract*—This paper presents a linear information-theoretic model for neural signal processing using Chinese philosophical ideas, in particular Yin and Yang. The main goal is to provide a mathematical model that can explain neural activity in accordance with concepts in traditional Chinese medicine. As yet, the lack of such models has prevented a more formal explanation of the efficacy of traditional treatments related to the nervous system, such as acupuncture. According to the proposed model, a synapse performs a linear operation on its input. This operation, when plotted in polar coordinates, follows the shape of a Yin-Yang symbol. Synaptic learning becomes synonymous with adapting the size and rotation of the Yin-Yang symbol. The model distinguishes between the perceived input and the actual input. Both, perception and reality, coincide in the golden ratio. Learning thus becomes a process of aligning perception with reality, which is a novel learning concept.**

I. Introduction

Chinese philosophical concepts play a major role in traditional Chinese medicine. However, most major concepts have eluded a closer mathematical investigation so far. Using earlier results in [1], this paper shows that one of the most important concepts, namely Yin and Yang, is not an obscure philosophical idea, but that it has a well-defined mathematical meaning. Such a formalization of philosophical ideas is important in order to give Chinese medicine a strong theoretical foundation, which would also increase its acceptance in the western world. As an example for such a formalization, this paper proposes an information-theoretic model of neural activity in synapses and neurons, using the mathematical definition of the Yin-Yang symbol. To get a better understanding of traditional treatments, such as acupuncture, better models of neural activity are needed. In the view of many western researchers, traditional Chinese medicine is lacking proper theoretical models to explain the efficacy of treatments. Of course, Chinese researchers may not agree with this viewpoint, and argue that there are models available, alas not very mathematical ones. Models that are both mathematical and that use Chinese concepts may help solve this dilemma. After all, it is only natural to use a neural model based on Chinese philosophical principles to explain the efficacy of medical methods in traditional Chinese medicine. As such a model, the proposed model offers an explanation for neural activity and learning, all in accordance with traditional Chinese concepts. In fact, according to the model, a neural learning process is a linear operation on the Yin-Yang symbol.

The paper structure is as follows: Section II shows how to render the Yin-Yang symbol properly according to its original meaning. This section is taken from [1]. Section III shows how the Yin-Yang symbol can be closely approximated with a linear information model. Then, Section IV presents the information-theoretic model underlying the synaptic signal processing as it is assumed in this paper. Section V describes how learning works for the proposed model and explains the difference between perceived input and actual input, including the role of the golden ratio. Finally, Section VI discusses the application of the proposed model to traditional Chinese medicine, and acupuncture in particular, and a summary concludes the paper.

II. Yin and Yang

According to Chinese philosophy, there are two opposing forces in the world, Yin and Yang [2], [3]. Yin and Yang are not only believed to be the foundation of our universe, but also to flow through and affect every being. Typical Yin-Yang opposites are for example night/day, cold/hot, rest/activity. Figure 1 shows the well-known black-and-white symbol of Yin and Yang. We can see two intertwining spiral-like curves

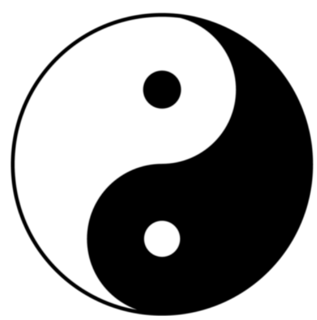

Fig. 1. A common Yin-Yang symbol.

in Figure 1, which are actually semicircles in this simplified graphics, separating the Yin and Yang area. The small spots of different color in each area indicate that both Yin and Yang carry the seed of their opposites; Yin cannot exist without Yang, and Yang cannot exist without Yin. These spots will play no role in this chapter. Neither will the assignment of black and white to Yin and Yang have any significance here, though Yin is typically associated with black and Yang with

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white. Contemporary literature has been mostly neglecting the plotting of the Yin-Yang symbol, paying more attention to philosophical questions. It turns out that the original Yin-Yang symbol is more complex than its modern representation as two semicircles suggests [4], [5]. The Yin-Yang symbol has its origin in the I-Ching; one of the oldest and most fundamental books in Chinese philosophy [6], [7]. The Yin-Yang symbol is tightly connected with the annual cycle of the earth around the sun, and the four seasons resulting from it. To investigate this cycle, the ancient Chinese used a pole that they put up orthogonally to the ground, as shown in Figure 2. With this setup, the ancient Chinese were able to record precisely the positions of the sun's shadow and divide the year into different sections. They measured the shortest shadow during the summer solstice, and measured the longest shadow during the winter solstice. After connecting the measured points and dimming the part that reaches from summer solstice to winter solstice (Yin), they arrived at a chart like the one in Figure 3. The resemblance between this chart and the modern Yin-Yang symbol in Figure 1 is striking. Figure 3 provides visual evidence that the original Yin-Yang symbol describes the change of a pole's shadow length during a year. In fact, by rotating the chart and positioning the winter solstice at the bottom, the Yin-Yang chart of the ancient Chinese becomes very similar to the modern Yin-Yang symbol depicted in Figure 1. The white area of the Yin-Yang symbol is typically called Yang. It begins at the winter solstice and indicates a beginning dominance of daylight over darkness, which is the reason why the ancient Chinese associated it with the sun (or male). Accordingly, the dark area of the Yin-Yang symbol represents Yin, which begins with the summer solstice. Yin indicates a beginning dominance of darkness over daylight. The ancient Chinese therefore associated it with the moon (or female).

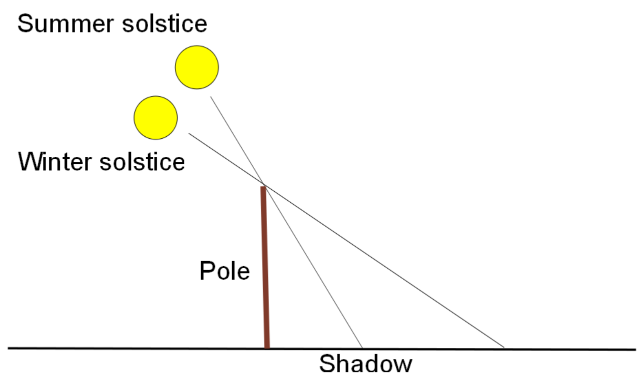

Fig. 2. Shadow model.

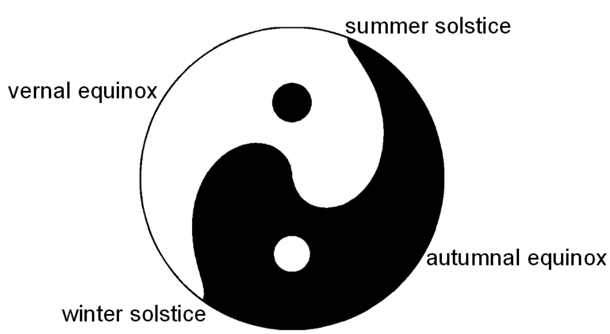

Fig. 3. Yin-Yang symbol for Latitude $L = 68°$ (near polar circle) with equinoxes and solstices.

The rendering method for the Yin-Yang symbol presented here is based on daylight hours, which are connected with shadow lengths [1]. A long day has the sun standing high on the horizon at noon, casting a short shadow. On the other hand, a short day is the result of the sun standing low on the horizon at noon, which in turn produces a long shadow. For computing the daylight time for a given day in the year, this section uses the formula given in [8], [1]. The formula takes many different factors into account, most notably the refraction of the earth's atmosphere. The daylight model presented here is therefore an accurate description of the actual daylight measurement of an observer on the ground. A detailed investigation of the formula is beyond the scope of this paper, though. The formula requires two input parameters, namely the day J of the year and the latitude L of the observer's location. It consists of three parts. The first part computes an intermediate result P, which is the input to the second part D', which in turn is input to the third part D that provides the final result. The equation for the first part is:

$$\begin{aligned} P &= \arcsin[0.39795 * \cos(0.2163108 + 2 * \arctan\{... \\ &\quad ...\ 0.9671396 * \tan[0.00860(J - 186)]\})] \end{aligned} \tag{1}$$

Given P, the second and third part then compute the actual day length D in terms of sunshine hours as follows:

$$D' = \arccos\left\{ \frac{\sin\left(\frac{0.8333*\pi}{180}\right) + \sin\left(\frac{L*\pi}{180}\right) * \sin(P)}{\cos\left(\frac{L*\pi}{180}\right) * \cos(P)} \right\} \tag{2}$$

$$D = 24 - \left(\frac{24}{\pi}\right) * D' \tag{3}$$

Using these equations, Figure 4 shows the daylight time for each day of the year and a latitude of 68°. This latitude is close to the polar circle, or Arctic Circle, in the northern hemisphere. The equivalent latitude in the southern hemisphere is the Antarctic Circle. The Arctic Circle marks the southernmost latitude in the northern hemisphere where the sun shines for 24 hours at least once per year (midnight sun) and does not shine at all at least once per year. Theoretically, the Arctic Circle marks the area where these events occur exactly once per year, namely during the summer and winter solstices. However, due to atmospheric refractions and because the sun is a disk rather

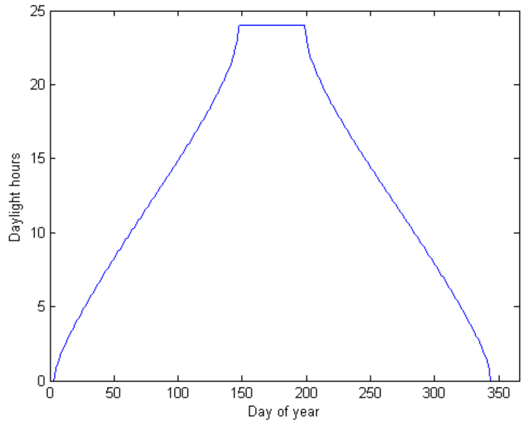

Fig. 4. Daylight hours for Latitude $L = 68°$ (near polar circle).

than a point, the actual observation at the Arctic Circle is different. For example, the midnight sun can be seen south of the Arctic Circle during the summer solstice. According to Figure 4, the midnight sun shines for about 50 days at latitudes around $68°$. Figure 5 shows the daylight hours in Figure 4 as a polar plot. In this polar plot, the distance to the origin stands

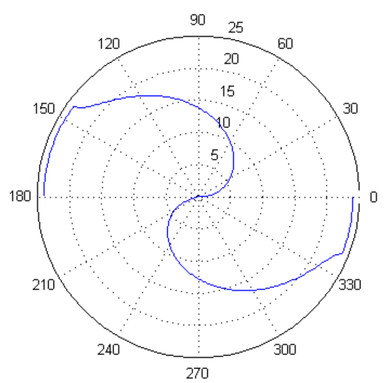

Fig. 5. Polar daylight plot for Latitude $L = 68°$ (near polar circle).

for the daily sunshine hours. One full turn of $360°$ corresponds to one year. There is another important difference to Figure 4, though. For the second half of the year, Figure 5 shows the hours of darkness instead of the daylight hours. The number of hours with darkness is simply the number of daylight hours subtracted from 24. Drawing the daylight hours in such a way produces the two spirals depicted in Figure 5. Coloring the areas delimited by both spirals and the outer circle in black and white then produces a rotated version of the Yin-Yang symbol in Figure 3. For latitudes around the polar circle, the spirals in Figure 5 originate either directly in the origin of the polar plot or in a point close to it. This is because there will be at least one day with no sunshine.

Figure 6 shows more examples of Yin-Yang symbols generated with the daylight model for L = $70°$, L = $75°$, and L = $80°$. All polar plots are rotated counter-clockwise by $45°$ so the x-axis is vertical. Both spots in each Yin-Yang symbol lie on the vertical axis, plotted halfway between the polar plot's origin and the outer circle.

Note that for latitudes L with $|L| \leq 68°$, the Yin-Yang symbol will look similar to the symbols observed at the polar circles when plotted in the following way: Instead of the daylight hours, the polar plot shows the daylight hours minus the minimum annual day length. Furthermore, instead of the number of hours with darkness, the polar plot shows the difference between the maximum annual day length and the number of daylight hours.

III. Information-Theoretical Yin-Yang Model

The mathematical formulation of the Yin-Yang symbol given in Eq. 1, Eq. 2, and Eq. 3 is clumsy. This section presents a more concise description of the Yin-Yang symbol. It shows that a linear information-theoretic model can approximate the Yin-Yang symbol with an average error of less than 1% with respect to the day length. To measure information, the paper uses the standard Shannon way, where information is measured using the negative binary logarithm ($-\log_2(p)$) for a probability value p [9]. The expected information is then simply the product $-p \cdot \log_2(p)$. The model proposed here has the following form, where $\Theta(p)$ and $r(p)$ are the angular and radial coordinates, respectively, and $p \in [0.5, 1]$ is the model's input.

$$\Theta(p) = -\pi * \log_2(p) \tag{4}$$
$$r(p) = -24 * \log_2(p) \tag{5}$$

Both equations can be summarized in one equation:

$$\Theta(p) = \frac{\pi}{24} * r(p) \tag{6}$$

Figure 7 shows the approximation of one branch of the Yin-Yang symbol for $L = 68°$ (red curve), which we obtain when applying Eq. 6. The figure also shows the original branch computed with the equations 1, 2, and 3 (blue curve). We

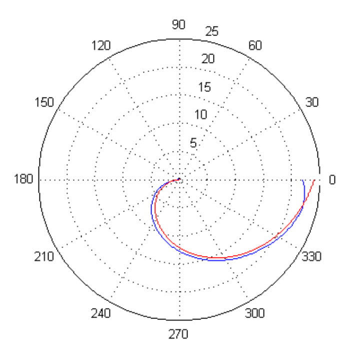

Fig. 7. Approximation of one branch of the Yin-Yang symbol for $L = 68°$ (close to polar circle) with a linear information model (red).

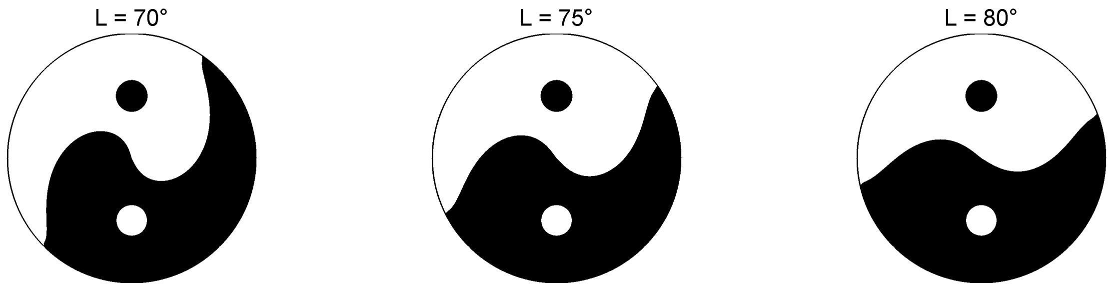

Fig. 6. Yin-Yang symbols for L = 70°, L = 75°, and L = 80°.

see that this approximation of the Yin-Yang symbol already provides a very close model of the Yin-Yang symbol. The model given by Eq. 6 can be further improved by using a linear regression, exploiting the linear relationship between the angular and the radial coordinate. Let

$$\Theta(p) = mr(p) + c \tag{7}$$

be the general form of the linear information model for the Yin-Yang symbol, where m is the slope, c is the offset or intercept, and $r(p)$ is a logarithmic function of the input. Accordingly, the linear approximation has the form $\Theta' = m'r + c'$, where m' and c' are the slope and intercept of the regression line, respectively. Figure 8 shows the optimal regression line (red) obtained for the Yin-Yang branch (blue) shown in Figure 7. Note that the range of the angular coordinate is $[\pi, 2\pi]$ and the radial coordinate has been normalized to $[0, 1]$, showing the normalized day length with the daylight hours of each day divided by 24. As we can see in Figure 8, the red regression line provides an almost perfect fit. Only toward the limits of the angular range does the linear information model differ from the original Yin-Yang model. This is mainly due to numerical problems of the approximated function. With $m' \approx 0.134$ and $c' \approx 3$, the mathematical equation for the regression line is

$$\Theta'(p) = 0.134r(p) + 3 \tag{8}$$

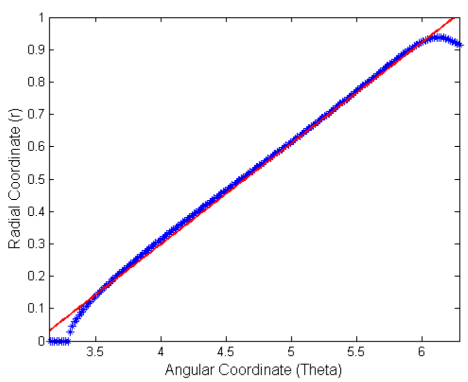

Fig. 8. Linear regression line (red) for a branch (blue) of the Yin-Yang symbol at $L = 68°$ (close to polar circle).

Or, in another form stressing that Θ' is indeed a linear function of information:

$$\Theta'(p) = 3 - 3.2 \cdot \log_2(p) \tag{9}$$

The median error of this model is 0.23, which is less than 1% with respect to a day length of 24 hours. Figure 9 shows a close-up of all approximations in one figure. The original Yin-Yang branch is again shown in blue. The first rough approximation of Eq. 6 is shown in green, and the optimal linear regression model is shown in red. We can see that the linear regression model lies between the rough approximation (green) and the Yin-Yang branch (blue), and we can see that it is closer to the Yin-Yang branch.

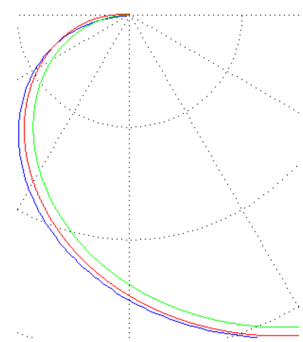

Fig. 9. Approximation of one branch of the Yin-Yang symbol for $L = 68°$ (close to polar circle) with a linear model (green) and a linear regression model (red).

IV. Neural Model

The main building blocks of the nervous system are neurons, which are electrically excitable cells that can process and transmit information by electrical and chemical signaling. Neurons connect to each other via synapses, which transmit signals from one neuron to another. All neurons and their connections together form a neural network. Figure 10 shows a typical signal path from one neuron to another. The lower right corner of Figure 10 shows a close-up of a synapse. Synaptic signals from other neurons are typically received via a neuron's soma and via its dendrites. The neuron sends

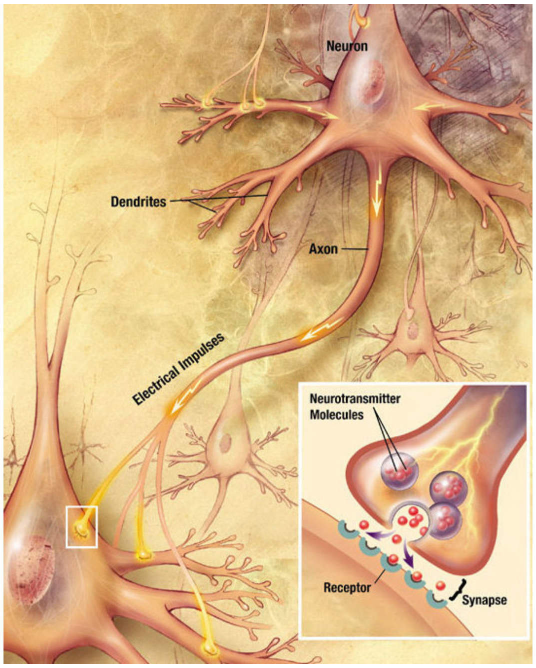

Fig. 10. A signal propagating down an axon to the cell body and dendrites of the next cell (Source: NIA/NIH).

signals to other cells via its axon, which is in turn connected to the soma and dendrites of other cells. A synapse is therefore the main contact between two neurons, connecting the axon of one neuron to the dendrites and soma of another. If the excitation received by a neuron exceeds a threshold, the neuron generates an action potential, which originates at the soma and propagates along the axon, activating the synapses of other neurons.

Researchers have tried to understand the neural signal processes, but our understanding is still limited. Today, there exist theoretical models that can partly explain the biological signal processes in the nervous system. One of the most successful ones has probably been the Hodgkin-Huxley model [10]. There have also been attempts to mimic human cognitive reasoning and to use artificial neural networks for practical pattern recognition applications [11], [12]. Different types of artificial neural networks have been proposed, and the multilayer feedforward networks with backpropagation learning are probably among the most successful [13], [14].

The linear information-theoretic model proposed here differs from other models in that it does not define a decision boundary, such as the Perceptron for instance [12]. Instead, it tries to learn the expected information (uncertainty) contained in a given input stimulus. It does so by adjusting, or training, its two main parameters, slope and intercept. The main idea is that training of the slope involves learning of the perceived stimulus, so that the perceived postsynaptic information equals the expected information of the perceived stimulus. Training of the intercept, on the other hand, ensures that the perceived postsynaptic information matches the actual presynaptic information of the true stimulus.

The proposed model uses the sigmoid function to process the input stimulus. The sigmoid function plays an important role in biological neural network models, as for instance in the Hodgkin-Huxley model [10]. This fact has motivated the use of the sigmoid function across a wide range of artificial neural networks, most notably the feedforward/backpropagation networks, where it plays the role of a transfer function [13], [14]. The sigmoid function $Sig(x)$ has the following mathematical form:

$$Sig(x) = \frac{1}{1+e^{-\lambda x}}, \tag{10}$$

where input x has no bound and parameter λ controls the steepness of the function. Figure 11 shows the sigmoid function for $\lambda = 1$, $\lambda = 2$, and $\lambda = 0.5$. In the Hodgkin-

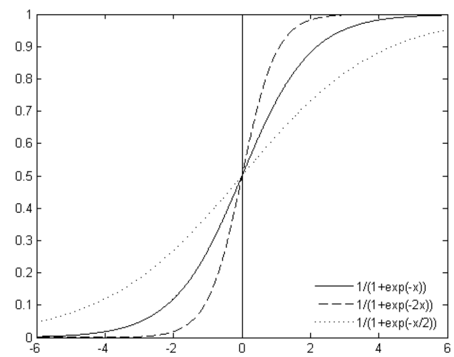

Fig. 11. Sigmoid function for $\lambda = 1$, $\lambda = 2$, and $\lambda = 0.5$.

Huxley model, the sigmoid function is related to the sodium conductance of a cell membrane, which is an important feature of the membrane's electrical behavior [10]. In particular, Hodgkin and Huxley assume that the sodium conductance is proportional to the number of specific molecules on the inside of the membrane but is independent of the number on the outside. From Boltzmann's principle the proportion P_i of the molecules on the inside of the membrane is related to the proportion on the outside,P_o, by

$$\frac{P_i}{P_o} = \exp[(w + zeE)/kT], \tag{11}$$

where E is the potential difference between the outside and the inside of the membrane, w is the work required to move the molecule from the inside to the outside of the membrane when $E = 0$, e is the absolute value of the electronic charge, z is the valency of the molecule (i.e. the number of positive electronic charges on it), k is Boltzmann's constant, and T is the absolute temperature [10]. With $P_i + P_o = 1$, the expression for P_i becomes

$$P_i = \frac{1}{1+\exp\left(-\frac{w+zeE}{kT}\right)} \tag{12}$$

This is the typical form of the sigmoid function, as shown in Eq. 10.

Now, for the basic sigmoid function in Eq. 10, with $x \geq 0$ and $\lambda = 1$, let $s(x) = e^{-x}$ be a stimulus with $s(x) \in [0, 1]$.

For a given input stimulus $s(x)$, the perceived input stimulus p is then given by

$$p(s) = Sig(s) = \frac{1}{1+s} \tag{13}$$

The sigmoid function thus maps the input range of the stimulus, which is $[0,1]$, to the perception range of our information-theoretic model, which is $[0.5,1]$.

The neural model proposed in this paper is modeled on the information-theoretic Yin-Yang model developed in Section III. Note that the angular coordinate in the linear Yin-Yang model is actually information. Thus, the Yin-Yang model is a linear model in which input and output are both information in the Shannon sense (Eq. 7 and Eq. 9). The angular identifier Θ will therefore be replaced with the identifier I here. The proposed neural model then becomes

$$\begin{aligned} I &= -m \cdot \log_2\left(\frac{1}{1+s}\right) + c & (14) \\ &= -m \cdot \log_2(p) + c & (15) \end{aligned}$$

The main assumption is that each synapse of a neuron transmit signals according to Eq. 15. The neuron itself then integrates over all outputs of its synapses and sends the result to other neurons via its axon. A neuron thus essentially computes the entropy of its input. The model postulates that adjusting the slope m involves learning of the perceived stimulus, i.e. $m = 1/(1+s)$, so that Eq. 15 computes the expected information for the perceived stimulus plus a constant c. The next section discusses the meaning of this constant according to the information-theoretic model proposed here.

V. Golden Ratio

Modeling the behavior of synapses has long been in the focus of machine learning, as synapses are suspected to be one the main locations where learning and adaptation takes place. This paper assumes that a synapse performs a linear operation on its input, according to the definition in Eq. 15. Learning thus consists of adjusting the parameters of the linear model, which are slope and intercept. As outlined in the previous section, learning of the slope involves adjusting the slope to the perceived value p, so that $m = p$. Ideally, the perceived value p is also identical to the original stimulus s, which guarantees that the perceived uncertainty $-p \cdot \log_2(p)$ is equal to the original uncertainty of the actual stimulus s, which is $-s \cdot \log_2(s)$. With $m = p$, this is the case when m satisfies the following requirement:

$$\begin{aligned} m &= \frac{1}{1+m} & (16) \\ \Longrightarrow \quad m &= \frac{\sqrt{5}-1}{2} \text{ or } \frac{-\sqrt{5}-1}{2} & (17) \\ \Longrightarrow \quad m &\approx 0.618 \text{ or } -1.618 & (18) \end{aligned}$$

According to Eq. 18, our perception p equals the true stimulus s for $p \approx 0.618$. This is the golden ratio, or strictly speaking the reciprocal Φ of the golden ratio, which is typically symbolized by $\varphi \approx 1.618$ [15], [16]. The golden ratio motivates the learning of the intercept c in the information-theoretic model defined by Eq. 15. The idea is to move the linear model into the golden ratio so to speak, so that perception and reality coincide. This is the case when c is defined as follows:

$$c = m \cdot \log_2(p) - s \cdot \log_2(s), \tag{19}$$

For an intercept c defined according to Eq. 19, the expected presynaptic information matches the perceived postsynaptic information. Or, in other words, the input uncertainty matches the output uncertainty of the model. Deviating from the golden ratio blurs the input stimulus in the sense that the perceived stimulus no longer corresponds to the true stimulus. For a stimulus s equal to the golden ratio, learning of the intercept c is not necessary because for $m = p = s \approx 0.618$ the intercept will be zero ($c = 0$). This could potentially explain why the golden ratio is often preferred over other ratios [15]. According to the synaptic learning model presented here, the golden ratio can be perceived faster as it requires less learning. Similar to other linear neural models presented in the literature, the intercept c can act as an adaptive threshold that prevents action potentials for input signals that are too small. Furthermore, the model proposed here is supported by recent publications, which confirm that the golden ratio indeed plays a role in neural signal processing, e.g. [17], [18].

VI. Application to Chinese Medicine

Acupuncture is one of the most successful treatment methods for pain and disease in traditional Chinese medicine. The treatment involves inserting thin needles into a patient's skin. According to traditional Chinese medicine, these needles can correct imbalances or congestions in the flow of Qi, or life energy, through channels known as meridians. Western researchers have come up with different, and more mundane, theories of why acupuncture helps reduce pain. One of the most successful theories is the Gate Theory of pain proposed by R. Molzack and P. Wall [19], [20]. According to this theory, pain impulses are not directly transmitted from the body to the brain. They have to pass through a gate in the spinal cord, where they can be inhibited or blocked by inhibitory input from other nerve fibers. Nevertheless, it is fair to say that the principle of acupuncture is only partly understood to date. For example, acupuncture excites the nerve fibers for a short period of time, but the effects of acupuncture persist for a long time after the acupuncture needles have been removed. In its current form, the Gate Theory cannot explain this prolonged effect.

The proposed model defined by Eq. 15 is capable of both long-term learning and signal inhibition. Long-term, or slowly changing, parameters can be saved in the slope m and the intercept c of the model. Furthermore, the signal output can be inhibited by reducing either the slope or changing the intercept appropriately. According to the equations 6, 9, and 15, changing these parameters has a direct geometrical interpretation with respect to the Yin-Yang symbol. Changing the slope m is equivalent to changing the radius, and thus the size, of the Yin-Yang symbol. This becomes evident when we look at Eq. 6, which uses the radius given by the daylight model presented above, namely 24 hours. On the other hand, changing the intercept c, which means adding a different constant offset to the output, translates to a rotation of the Yin-Yang symbol. The proposed model therefore provides a geometric interpretation for learning processes that are intimately connected with the Yin-Yang symbol.

The proposed model implies that Qi is essentially information in the Shannon sense. Furthermore, according to the proposed model, the flow of Qi can be controlled at the synapses. Harmony and balance can be achieved by setting the parameters for a synapse properly. A distorted flow of Qi can be corrected by adjusting the slope and the intercept. Any adjustment of these parameters is automatically an adjustment of the Yin and Yang forces that determine the behavior of the synapse. These results could lead to mathematical, and yet very traditional, explanations of the efficacy of acupuncture in the future.

VII. Conclusion

The work presented in this paper shows that it is possible to formalize Chinese philosophical concepts. Using the daylight model for the Yin-Yang symbol, the proposed neural model incorporates an essential part of Chinese philosophy. According to the model, a synapse computes the expected information contained in a stimulus. A neuron then integrates the output of all its synapses and forwards the result, which is basically entropy, to other neurons. The paper claims that synapses perform linear operations and that they are capable of learning the model parameters, which means slope and intercept. Furthermore, the model distinguishes between the perceived stimulus and the actual stimulus, which can differ from each other. Learning therefore involves matching the expected information of the perceived stimulus with the expected information of the actual stimulus. The paper shows that the perceived and actual stimuli coincide when we observe the golden ratio. For this reason, the paper postulates that the golden ratio plays an important role in neural processes. Recent publications support this assumption. Moreover, the paper shows that learning is equivalent to adjusting the size (radius) and rotation of the Yin-Yang symbol. The hope is that the proposed model helps to explain the efficacy of traditional treatments, in particular acupuncture. It may also help to bridge the gap between western and eastern medicine. Future steps include the verification of the proposed model with biological reality, and the application to practical pattern recognition problems.

Acknowledgment

This research is supported by the Intramural Research Program of the National Institutes of Health (NIH), National Library of Medicine (NLM), and Lister Hill National Center for Biomedical Communications (LHNCBC).

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