Similarity Measurement Using Polygon Curve Representation and Fourier Descriptors for Shape-based Vertebral Image Retrieval

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ABSTRACT

Shape-based retrieval of vertebral x-ray images is a challenging task because of high similarity among the vertebral shapes. Most techniques, such as global shape properties or scale space filtering, lose or fail to detect local details. As the result of this shortfall, the number of retrieved images is so high that the retrieval result is sometimes meaningless. To retrieve a small number of best matched images, shape representation and similarity measurement techniques must distinguish shapes with minor variations. The main challenge of shape-based retrieval is to define a shape representation method that is invariant with respect to rotation, translation, scaling, and the curve starting point shift. In this research, a polygon curve evolution technique was developed for smoothing polygon curves and reducing the number of data points while preserving the significant pathology of the shape. The x and y coordinates of the simplified boundary points were then converted into a bend angle versus normalized curvature length function to represent the curve. Finally, the Fourier descriptors of the shape representation were calculated for similarity measurement. This approach meets the invariance requirements and has been proved to be efficient and accurate.

Keywords: Shape Representation, Shape Similarity, Vertebral Image, Shape-based Image Retrieval, Curvature Function, Fourier Descriptors, X-ray Image

1. INTRODUCTION

Manual indexing and retrieval for biomedical content such as x-ray images from a large image database is a prohibitively labor intensive task. An automated or computer-aided retrieval system will greatly improve the image retrieval process. Medical images, especially images created by digitizing film x-rays of human cervical and lumbar spines, generally have low contrast and low image quality. Traditional image matching techniques based on grayscale image distance or correlation are computationally expensive and not meaningful for the retrieval of x-ray images. Shape-based techniques are more suitable than grayscale- or feature-based techniques for this task. A completed shape-based image retrieval system includes contour noise removal, shape representation, similarity measurement, and fast indexing. The biggest challenge of shape-based retrieval is to mathematically describe the shapes and to derive a similarity measurement to compare the shapes.

Shape-based retrieval of vertebral x-ray images is a challenging task because of highly similar shape properties. Figure 1 shows a cervical x-ray image with a shape contour outlined in blue. The shape representation methods need to work with planar closed curves as shown in Figure 1. While reducing the number of redundant data points to minimize the computation requirement, the shape representation method should also preserve local details so that highly similar shapes can still be distinguished and ranked. Because x-ray images are not always taken in a fixed position, the shape representation methods should also represent the shape in a geometrical invariant manner so that the similarity between two shapes can be calculated disregard the variations in rotation, translation, and scaling. Moreover, shape data are extracted from a grayscale image and recorded as a sequence of x and y coordinates. Another important requirement for an efficient shape representation method is that the starting point shift of the curve should not have any effects on similarity measurement.

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There are many different techniques to describe the shapes for content-based image retrieval applications. Global shape properties such as size, perimeter, convex perimeter, elongation, roughness, and compactness, etc., can be used for measuring similarity [1]. Invariant moments have also been used for discriminating shapes [2]. Multi-stage modification using invariant moments has shown very good results [3]. Other methods using higher order moments include generalized complex moments [4] and Zernike moments [5]. Multi-scale shape representation has been used to smooth and simplify the contours [6-8]. The curvature function was then used to analyze the smoothed curve in order to determine the critical points on the curve for comparing curve segments or token. Another polygon curve representation method is done in the tangent space, also called turn function. This method uses curve evolution to remove small variations and less significant features and then represents the curve in tangent space [9-11]. Shapes or contour points can also be described in frequency domain [12-13]. The contour points of a polygon must be represented using turn function or bend angle function so that they are invariant to translation, rotation, and scaling. Fourier descriptors can then be used to measure shape similarity [14] because it is invariant to starting point shift of the polygon curve.

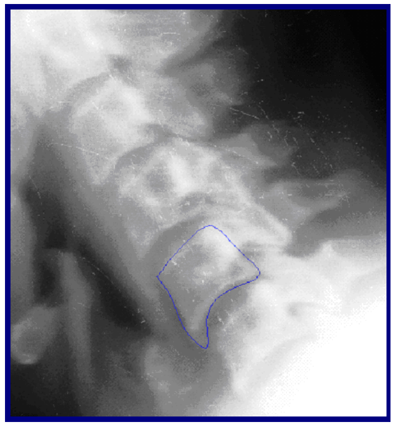


Figure 1. A cervical contour.

The input to the shape representation algorithms is a collection of boundary points recorded as a sequence of x and y coordinates. The shape contours must be extracted from the database image before building the database for indexing. Most techniques such as global shape properties, scale space filtering lose or fail to detect the local details. As the result of it, the number of retrieved images is so high that the retrieval result is sometimes meaningless. Shape representation and similarity measurement techniques for shape indexing must be able to distinguish shapes with minor variations. Figure 2 shows four typical vertebral contours with very similar shape properties.

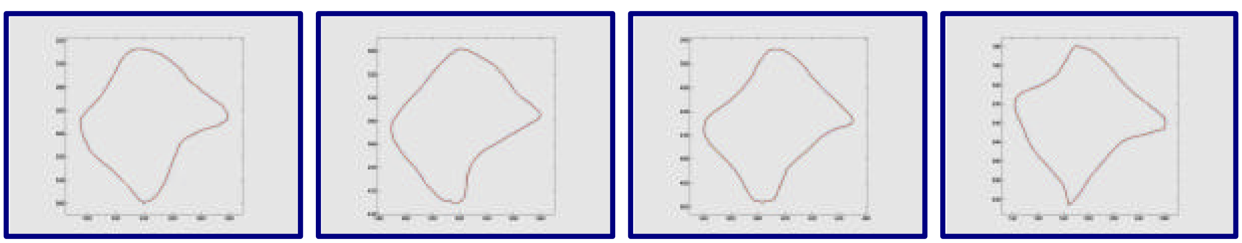


Figure 2. Four vertebral contours with similar shape.

In this paper, a new polygon curve evolution technique was developed for smoothing polygon curves and reducing number of data points while preserving the significant pathology of the shape. The x and y coordinates of the smoothed boundary points were then converted into a bend angle verses normalized curvature length function to represent the curve. Finally, the Fourier descriptors of the shape representation were calculated for similarity measurement. Shape similarity measure was designed so that the input shapes can be compared against the shapes in the database and the similarity scores can be obtained to evaluate the performance. Twenty randomly selected shapes were used for evaluation. Shapes that are similar were grouped together and categorized. The performance was evaluated by examining the number of retrievals in the same group. In this paper, the pre-procession and shape representation techniques will be discussed in Section 2. Section 3 describes the Fourier descriptors and similarity measurement. Data and result will be analyzed in Section 4 and the conclusions and future work will be presented in Section 5.

II. CURVE EVOLUTION AND SHAPE REPRESENTATION

Polygon curves were generated by segmenting the x-ray images and recorded the results in a sequence of x and y coordinates. Many data points on the contour are redundant or edge noise and must be removed. Curve evolution has been used to describe shapes in different levels of detail [9-11]. A modified version of curve evolution technique has been developed and implemented to eliminate insignificant shape features such as short straight line segments.

2.1 Curve Evolution

Curve evolution has been used to reduce the influence of noise and to simplify the shapes by removing irrelevant and keeping relevant shape features. This was achieved by iteratively comparing the relevance measure of all vertices on the polygon. Higher relevance value means that the vertex has larger contribution to the shape of the curve. For each of these iterations, the vertex that has the lowest relevance measure was removed and a new segment was established by connecting the two adjacent vertices. The relevance measure can be expressed and calculated as

$$K(s_1, s_2) = \frac{b(s_1, s_2) l(s_1) l(s_2)}{l(s_1) + l(s_2)} \qquad \text{Equation 1}$$

, where ß is the turn angle and *l* is the normalized length. The relevance measure is in direct proportion to the turn angle and the length of the curve segment. As illustrated in Equation 1, Figure 3 (a) shows a vertex that has lower relevance measure than (b) because of shorter length and has lower relevance measure than (c) because of smaller turn angle.

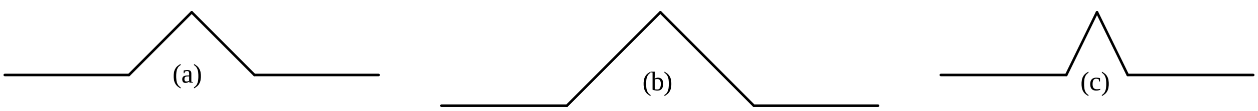
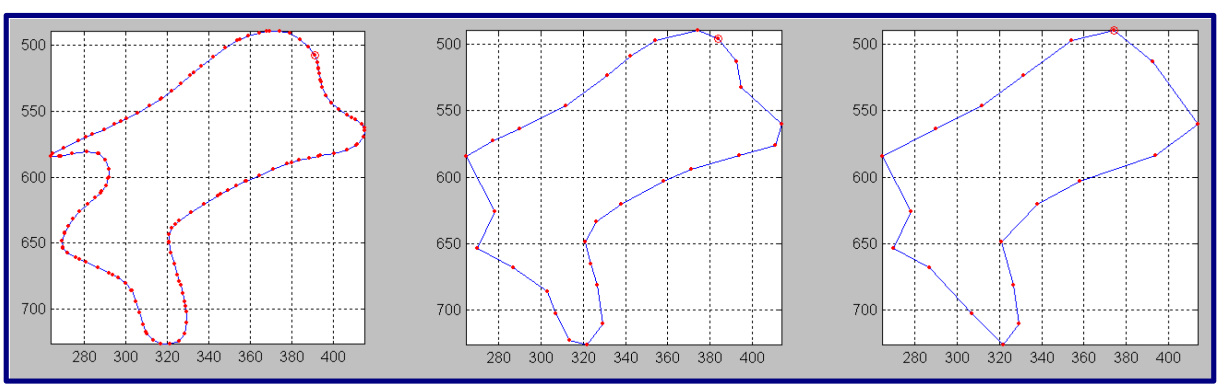


Figure 3. Vertices with different relevance measure.

Equation 1, while works well for describing shapes in different levels of detail, starts losing the significant pathology of the shapes as the number of data points decreases. Figure 4 shows the results of curve evolution using Equation 1. The original shape contour has 172 data points as shown in Figure 4 (a). It was reduced to 30 points and 20 points using Equation 1 as shown in Figures 4 (b) and 4 (c), respectively. Several critical points were lost when the number of data points was reduced to 30 (Figure 4 (b)). The situation got even worse when the number of data points was reduced to 20 points (Figure 4 (c)). After removing vertices with low relevance measure using Equation 1, the remaining vertices can hardly represent the original shape correctly.



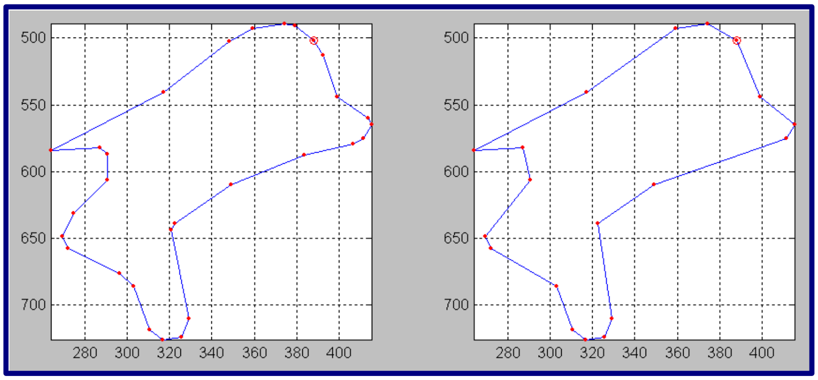
(a) (b) (c)

Figure 4. (a) Original contour with 172 data points, reduced to (b) 30 points (b) and (c) 20 points using Equation 1.

A new relevance measure equation was developed to remove short and straight line segments so that the critical points can be detected and preserved. This new curve evolution technique effectively reduces the data points and keeps significant shape features. It removes the vertices that have short length and/or their turn angles are close to 180 degrees (straight line). Equation 1 was modified and expressed in Equation 2 in order to achieve this task.

$$K(s_1, s_2) = \frac{|(b(s_1, s_2) - 180)|\, l(s_1) l(s_2)}{l(s_1) + l(s_2)} \qquad \text{Equation 2}$$

As shown in Figure 5, Equation 2 keeps data points that are critical to preserving detail pathology of the shape. The critical vertices that represent the concave and convex points of the shape were preserved even when the number of data points was reduced to 20 points (Figure 5 (b). More local details were preserved using Equation 2.

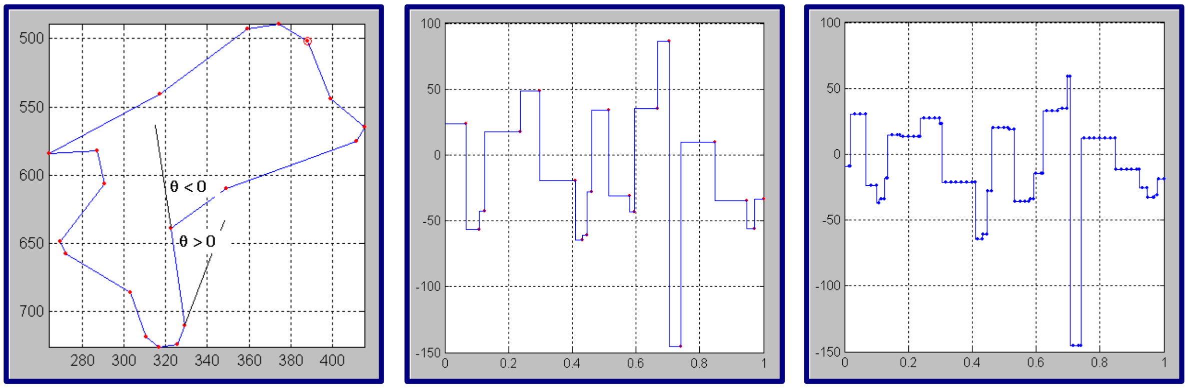


(a) (b)

Figure 5. Number of data points reduced to (a) 30 points and (b) 20 points using Equation 2.

2.2 Shape Representation

Because x-ray images could be taken from different angles and locations and their size varies, the shape representation must be invariant to translation, rotation, and scaling. The bend angle was calculated so that the clockwise turn gives a negative angle whereas a counter clockwise turn gives a positive angle as shown in Figure 6 (a). This method represents a closed polygon curve *C (m vertices)* as $T(l)$, bend angle as a function of normalized accumulated length *l*. Because it does not contain orientation information, this representation meets the rotation invariant requirement. Normalized length makes it independent to the polygon size so that it meets scaling invariant requirement. Figure 6 (b) shows the function $T(l)$ of a 20-point bend angle function. The x-axis represents the normalized length. The only problem left is the starting point shift invariant requirement and it can be taken care of by the shift invariant



(a) (b) (c)

Figure 6. (a) Bend angle, (b) bend angle vs. normalized length, and (c) data samples for similarity measurement.

property of the power spectrum. Data used for calculating the power spectrum of the bend angle function $T(l)$ were sampled at a fixed length interval. For example, if twenty data points are used for calculating the power spectrum, then the data will be sampled at an interval of 0.05 of the total length. Figure 6 (c) shows the sampling data of the bend angle function for calculating Fourier Descriptors for similarity measurement.

III. FOURIER DESCRIPTORS AND SIMILARITY MEASUREMENT

In this section, detail algorithms for calculating Fourier Descriptors will be discussed. The similarity measure for comparing Fourier Descriptors is also included in this section.

3.1 Fourier Descriptors

The Fourier expansion of $T(l)$ is expressed as .

$$\Theta(l) = m_0 + \sum_{n=1}^{\infty} (a_n \cos nl + b_n \sin nl) \qquad \text{Equation 3}$$

, where a_n and b_n are coefficients for each frequency component. Since $T(l)$ is a step function, *μ_0, a_n, and b_n can be derived as*

$$m_0 = -p - \frac{1}{L}\sum_{k=1}^{m} l_k q_k$$

$$a_n = -\frac{1}{np}\sum_{k=1}^{m} q_k \sin\frac{2pn\,l_k}{L} \qquad b_n = \frac{1}{np}\sum_{k=1}^{m} q_k \cos\frac{2pn\,l_k}{L} \qquad \text{Equation 4}$$

$$\textit{where } \; l_k = \sum_{i=1}^{k} l_i \quad \textit{and} \quad L = \sum_{i=1}^{m} l_i = \textit{the total length}$$

The power spectrum of the bend angle function is invariant to the shift in length (*l* in this case). Because of this property, Fourier descriptors of a bend angle function (function of normalized length) meet all invariant requirements for shape description for shape-based retrieval. The power spectrum (A_n) and phase angle information (F_{jk}) can be calculated as follow [14]:

$$A_n = \sqrt{a_n^2 + b_n^2} \quad \textit{and} \quad a_n = \tan^{-1}(b_n / a_n)$$

$$F_{jk} = j^* a_k - k^* a_j \quad \textit{where } j^* = j / \gcd(j,k) \qquad \text{Equation 5}$$

$$\gcd : \textit{greatest common divisor}$$

3.2 Similarity Measurement

Similarity measures were derived based on the *l_2-norm* of the shape features. In other words, query is measured (scored) based on an appropriate distance measure in the feature space. This can be expressed as follow. Suppose the significant shape features are selected and recorded as a one-dimensional vector and are expressed as $S_A = [a_1, a_2, a_3, \cdots]$ and $S_B = [b_1, b_2, b_3, \cdots]$. The distance measure between two vectors in the feature space can be calculated as

$$\|d\|_2 = |S_A - S_B|^2 = \sqrt{\sum_{i=1}^{n} |a_i - b_i|^2} \qquad \text{Equation 6}$$

Based on Equation 6, the difference (dissimilarity) between two sets of Fourier descriptors can be measured as :

$$D_A[c,c'] = (\sum |A_k - A_k^{'}|^2)^{1/2} \quad \textit{for the amplitude or}$$

$$D_a[c,c'] = (\sum |F_{kj} - F_{kj}^{'}|^2)^{1/2} \quad \textit{for the phase angle}$$

Equation 7

,where A_k represents the amplitudes of the Fourier Descriptors and F_{kj}. The difference between the amplitudes (D_A) can be normalized and rescaled to between 0 and 100.

$$S_A[c,c'] = 100 - D_A \quad (\textit{where } D_A \in \{0,100\}$$

Equation 8

The similarity between two curves can then be measured as S_A=100-D_A so that a score of 100 denotes that the curves are identical and 0 means they are the least similar curves. Only D_A was used because the phase angle difference D_a contains the phase information that is not needed for calculating the similarity measure. Figure 7 shows the two shapes that were used for testing the Fourier descriptors algorithm. The small one (blue) is the original shape that is shown in Figure 4 (a) and the large one (red) is the same shape but was rotated by 20 degrees and rescaled by 1.2. The staring point of the data points on the contour was shifted by 10. Table 1 shows that the similarity measures stay constant even with all the above alterations. One of the data point was also moved to test the similarity measurement sensitivity. The similarity dropped to 98.3516 when one data point was moved by (20, 20).

Changes from the original	Similarity Measure
20 degree rotation	99.99999
1.2 scaling	99.99999
10 starting point shift	100.0
Combination of all three	99.99999
One point moved by (20,20)	98.3516

Table 1. Similarity measures after the alternations.

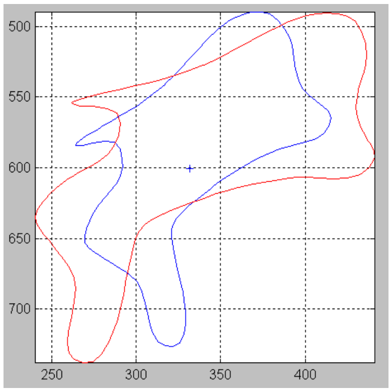
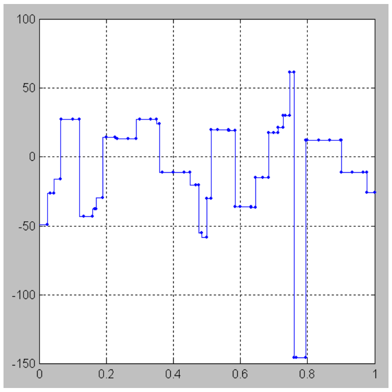
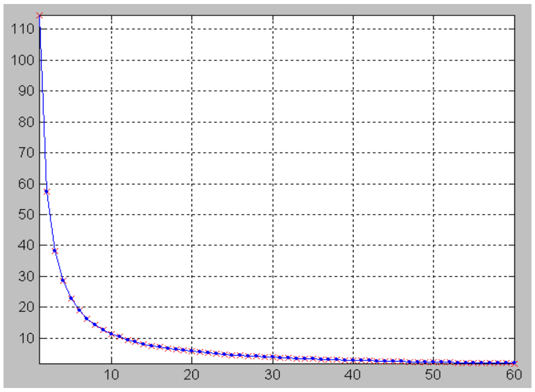


Figure 7. Test shape and the altered shape.

Figure 8 (a) shows the sampling data of the bend angle function of the altered shape (large). The two bend angle functions are almost identical because of the invariant properties of the bend angle function. The only difference between the two bend angle functions is the small shift in the x-axis (along the contour length) which was caused by the





(a) (b)

Figure 8 (a) Bend angle function and (b) Fourier descriptors.

starting point shift. The shift in the x-axis can be taken care of by the Fourier power spectrum's shift invariant property. Figure 8 (b) shows the Fourier descriptors of the two bend angle functions. Even with a starting point shift in the bend angle functions, the Fourier descriptors stayed the same.

IV. DATA ANALYSIS

The developed shape description and similarity measurement algorithms were tested on 20 selected vertebral image contours. The testing showed very promising results. Table 2 below shows the retrieval results represented as ranked similarity measures. The testing was performed by selecting one at a time of the twenty shapes as the inquiry and comparing it against the rest of the shapes in the database. In Table 2, Row 1 represents the shape that was chosen as the inquiry (from 1 to 20). Each column represents the retrieval result. Shapes with higher similarity measure were ranked higher and were placed closer to the top. For example, when using Shape #8 as the inquiry, Shape #7 had the highest similarity measure and was ranked the best match (to Shape #8). Shape #7 was followed by Shapes #6, #5, #9, and so on. Depending on the inquiry shape, the ranking was slightly different. However, the overall retrieval result was proved to be very accurate. Figure 9 shows the retrieval result with shape contours when Shape #8 was used as the inquiry.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
11	12	15	3	7	8	8	7	17	17	1	2	19	12	3	18	10	16	20	19
14	14	13	15	9	7	6	6	10	9	14	14	20	2	13	20	9	20	13	16
18	10	19	13	8	5	5	5	2	2	12	10	16	11	19	19	2	19	16	13
12	17	20	19	17	9	9	9	12	12	2	17	18	10	20	13	12	13	18	18
2	11	16	20	10	17	17	17	14	14	10	11	3	17	16	1	14	1	1	1
16	9	18	16	2	10	10	10	11	11	17	9	1	9	18	11	11	11	3	11
20	1	11	18	12	2	2	2	1	1	18	1	11	1	1	14	1	14	11	3
10	18	14	1	14	12	12	12	5	18	9	18	15	18	11	3	18	12	14	14
17	16	12	11	11	14	14	14	18	16	16	16	14	16	14	12	16	2	12	12
19	20	10	14	1	3	11	11	16	20	20	20	12	20	12	2	20	3	2	2
9	19	17	12	6	11	1	1	20	19	19	19	2	19	2	10	5	10	15	15
13	13	9	2	18	1	3	3	19	5	13	13	10	13	10	17	19	17	10	10
3	5	1	10	16	18	18	18	13	13	3	3	17	3	17	9	13	9	17	17
15	3	2	17	3	16	16	16	3	3	5	5	9	5	9	15	3	15	9	9
5	15	5	9	20	20	20	20	7	15	15	15	5	15	5	5	15	5	5	5
7	7	7	5	19	19	19	19	8	7	7	7	7	7	7	7	7	7	7	7
8	8	8	7	13	13	13	13	15	8	8	8	8	8	8	8	8	8	8	8
6	6	6	8	15	15	15	15	6	6	6	6	6	6	6	6	6	6	6	6
4	4	4	6	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4

Table 2. Retrieval results expressed as ranked similarity measures.

As shown in Table 2, shapes can be grouped together as being similar. Shapes #8, #7, #6, and #5 are very similar to one another and can be grouped together. The next close group consists of Shapes #9, #17, and #10. Other groups include (#2, #12, #14, and #11), (#1 and #3), and (#18, #16, #20, and #19). Shapes in the same group tented to stay very close together in the ranking. Shapes #4, #13, and #15 are very different from others. However, the algorithms did find Shape #3 as the closest match to Shape #4.

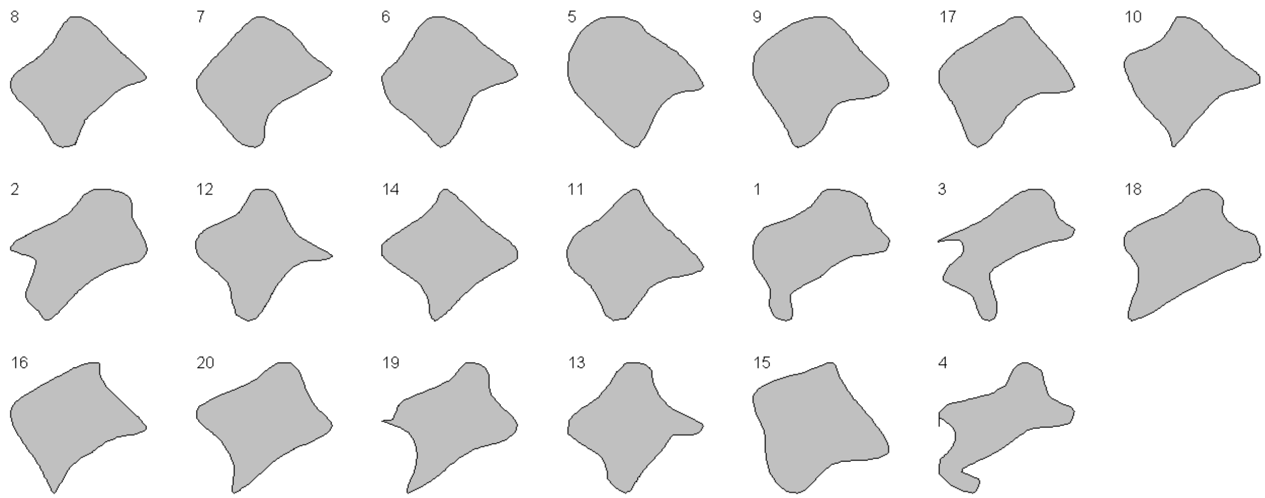


Figure 9. Retrieval result when Shape #8 was used as the inquiry.

V. CONCLUSIONS

In this research, a new curve evolution equation was derived and implemented to reduce the number of data points, to smooth the shape contour, and at the same time preserve the significant pathology of the shape. The bend angle verse the normalized length function was used to represent the shapes. This shape representation method meet the rotation, scaling, and translation invariant requirements for comparing shape contours extracted from vertebral x-ray images. Fourier descriptors were calculated to take care of the contour starting point shift problem so that the shape similarity could be measured. The l_2*-norm* was then calculated to measure the difference between two sets of Fourier descriptors to measure the difference between two shapes for similarity ranking. The testing has shown accurate result and has proved that shape-based vertebral x-ray image retrieval from an image database is feasible.

VI. ACKNOWLEDGEMENT

This research was supported in part by an appointment to the NLM Research Participation Program sponsored by the National Library of Medicine and administered by the Oak Ridge Institute for Science and Education.

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