# Exploring Surface Characteristics with Interactive Gaussian Images (A Case Study)

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## Abstract

The Gauss map projects surface normals to a unit sphere, providing a powerful visualization of the geometry of a graphical object. It can be used to predict visual events caused by changes in lighting, shading, and camera control. We present an interactive technique for portraying the Gauss map of polygonal models, mapping surface normals and the magnitudes of surface curvature using a spherical projection. Unlike other visualizations of surface curvature, we create our Gauss map directly from polygonal meshes without requiring any complex intermediate calculations of differential geometry. For anything other than simple shapes, surface information is densely mapped into the Gaussian normal image, inviting the use of visualization techniques to amplify and emphasize details hidden within the wealth of data. We present the use of interactive visualization tools such as brushing and linking to explore the surface properties of solid shapes. The Gauss map is shown to be simple to compute, easy to view dynamically, and effective at portraying important features of polygonal models.

**CR Categories:** I.3.8 [Computing Methodologies]: Computer Graphics—Applications I.3.5 [Computing Methodologies]: Computer Graphics—Computational Geometry and Object Modeling Curve, surface, solid, and object representations

**Keywords:**
Computational Geometry, Gauss map, Illumination and shading, Interactive visualization

## 1 INTRODUCTION

Rendering and modeling of complex objects often requires a deep understanding of the natural structure of surface shape. Surface curvature is often a primary characteristic used to describe local shape, drawing from the rich background of differential geometry, especially when considering smooth surfaces. Curvature has been explicitly illuminated using textures and glyphs to promote the understanding of surface shape [7]. Prinicipal curvature has also been used as an essential structuring element for non-photorealistic rendering [3]. The Gauss map, the projection of surface normals to a unit sphere, can be used to illuminate the natural structure of surface shape[1]. Graphics and vision researchers have used variations of Gauss maps as domains for comparing objects [5][11] and regularizing non-manifold surfaces [2]. This visualization of surface behavior has a complex relationship to the underlying model, suggesting the need for advanced visualization techniques. The application described here interactively displays two linked simultaneous views under user control; one is the model-view and the other is the Gauss-view. To aid the exploration, brushing is implemented permitting the user to focus on local patches of the model. Dynamic visualization of the Gauss map speeds understanding of complex surface properties.

IEEE Visualization 2002 Oct. 27 - Nov. 1, 2002, Boston, MA, USA
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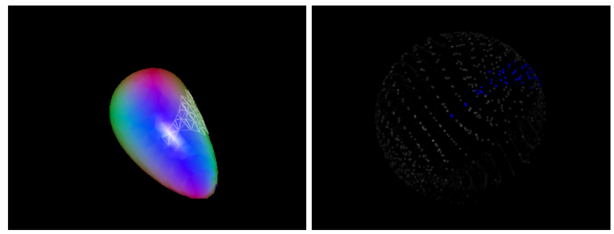

Figure 1: The Gauss Map – a projection of the normal of every point on an orientable surface to its corresponding locus on a unit sphere. In the illustration above, the aggregate normal for each selected triangle on the surface (left) is mapped to the Gaussian sphere and shown in blue (right).

## 2 THE GAUSS MAP

The Gauss map takes points on a 2–D manifold in $R^3$ and sends them to points on a unit sphere at the origin. Let $\vec{p} \in S$ be a point on a surface $S$. Also let the surface normal at $\vec{p}$ be $N(\vec{p}) = n_1 U_1(\vec{p}) + n_2 U_2(\vec{p}) + n_3 U_3(\vec{p})$ where $N(\vec{p})$ is the surface normal in the tangent space of $\vec{p}$ and $U_i(\vec{p})$ forms the natural frame for the tangent space at $\vec{p}$. Then the Gaussian map is $G(\vec{p}) := (n_1(\vec{p}), n_2(\vec{p}), n_3(\vec{p}))$. This mapping can be thought of as moving the normal of a point on a surface to a congruent parallel vector at the origin.

Consider a polygonal surface with face normals. Each point on the plane of a polygon will be mapped to the same point in the Gaussian image because each point has the same surface normal. Figure 1 show this type of mapping where each triangle is considered a sample. Instead of merely mapping a point to the sphere, one can place a weight on the sphere proportional to the surface area of the polygon. The weighted sphere can be viewed as the distributions of the surface normals and is referred to as the Extended Gaussian Image (EGI). The resulting EGI for a surface does not preserve spatial connectivity, since each face is mapped to a single point and no concerns about the edges and vertexes are taken into consideration. Despite this, the mapping for convex objects is injective. That is, if two convex objects mapped to the same EGI, then those objects must be congruent[9]. For non-convex objects, the EGI is not unique. Surfaces which are not congruent can map to the same EGI. Additionally, non-convex objects can occlude themselves, resulting in ambiguity in the EGI, where self-occluded surfaces will still be visible. For object identification Ikeuchi and Hebert use the observed EGI, which consist of only the surface on the object visible to the viewer [6].

Curvature can be thought of as the magnitude of the second derivative of a surface, indicating how fast the normal of the surface is changing. Gaussian curvature of a surface is the ratio of the change in the normal to the surface area, that is, $K = \frac{d\Omega}{dA}$ where $dA$ is a small patch on the surface and $d\Omega$ is the change in the normal [8].

For every point on a smoothly curved surface one can clearly apply the Gaussian mapping. Horn derives the relationship that the inverse of the absolute value of the Gaussian curvature for a smooth surface is the definition of the Extended Gaussian Image, which was derived by considering the following integration,

$$\int_{\Omega}\int \frac{1}{K} d\Omega = \int_{A}\int dA = A. \tag{1}$$

This relationship is critical for deriving other EGIs and interpreting the resulting images [4].

While there does not appear to be work on directly visualizing the EGI, previous papers have illustrated EGI to explaining the author's work. Horn uses an oriented histogram on a geodesic dome. Each cell of the histogram represents a discretized normal direction, with a glyph or color showing the number of normals in that direction. Tanaka illustrated the EGI of surface metrics by plotting samples on the Gaussian sphere[11]. In both cases, only normals are shown, with no indication of connectivity. In his book on Solid Shape, Koenderink describes the Gaussian image in terms of connectivity, describing surface properties in terms of folds and pleats of the mapping of surface normals [8]. Much of the inspiration for this work arises from his observations.

## 3 GAUSSIAN IMAGE OF A MESH

Mapping triangles in a mesh, rather than just isolated faces, helps the user understand the relationship between interesting areas on an EGI and the corresponding triangles. The mapping of triangles is a straightforward extension of the mapping of the faces. For each unique edge, the Gaussian image is taken of the two vertexes and then a line is drawn between them in the Gauss-view. If the surface is sufficiently sampled, a straight line can be drawn, otherwise the line needs to be a great arc of the unit sphere. Figure 2 shows a mesh of a bean-like shape (left) and its Gaussian image (center and right). Figure 3 shows the same mesh from another angle.

If all the model mesh triangles had the same area, one could use the size of the matching triangle in the Gaussian image to draw conclusion about curvature, but this is not always the case. If presented varying polygon size in the object mesh, curvature magnitudes can be calculated explicitly in order for this information to be made visible in the Gaussian image. There are two places where this information could be displayed, in the Gaussian mapped edges or in the triangles themselves. Since the area between the edges is large and the color of the edges is already used to express orientation, we chose to use the spaces between the vertices to portray additional curvature information.

## 4 SPLATTING THE EGI

We assume the surface to be smooth, continuous, and sufficiently sampled. What we wish to achieve is an EGI view which is smooth and clearly shows the curvature of the surface. The Phong shading model linearly interpolates vertex normal directions across the surface [10], resulting in the appearance of smoothly varying Gaussian curvature across each mesh triangle. Using an averaged linear curvature to color triangles in the Gauss-view would result in artifacts at sample boundaries. While these discontinuities are representative of the shaded appearance, they may give a misleading picture of the presumably smooth sampled surface. Instead, we wish each to map each sample (mesh triangle) into the Gaussian image as a distribution corresponding to the estimated distribution of normals associated with the sample. Using the Gaussian normal distribution as a reasonable approximation for the distribution of the normals for each triangle, we splat each sample onto the Gaussian sphere, scaled according to curvature [12].

The splatted EGI will be rendered as a texture mapped sphere, so the composition of the samples occurs in the texture map. For each sample, the splat kernal $h_v(d)$ has spread $\sigma$ corresponding to the standard deviation of vertex normals from the polygon normal $G(\vec{p})$. Specifically,

$$\sigma = \sum_{3} \frac{\rho}{3.0} \sqrt{\frac{1}{(N_i \cdot G(\vec{p}))^2} - 1} \tag{2}$$

where $N_i$ is the normal at the ith vertex and $\rho$ is a constant. The splat kernal $hn_v(d)$ is a function of distance in the texture map. Specifically,

$$d = \sqrt{\frac{1}{(P_{(s,t)} \cdot G(\vec{p}))^2} - 1} \tag{3}$$

where $P_{(s,t)}$ indicates coordinates on the texture map. Each splat is scaled by the triangle area $A_{\vec{p}} h_v(d)$. After all samples are composited, the resulting values are scaled to the range $[0,1]$.

We colormap the splatted values using a color scale with redundant hue and brightness variation. This scale has intuitive high and low values, as well as clear and natural progression from smallest to largest value. For additional flexibility, we provide a user-controlled bias function for the scalar range in order to provide additional detail in specific portions of the range. Figure 2(right) and 3(right) show the splatted EGI for two views of the bean shape.

## 5 INTERACTIVE GAUSSIAN IMAGES

Complex, non-convex surfaces can create confusing Gauss-views because multiple mesh points can map to the same location in the EGI. Additionally, the correspondence between points on a model mesh and points in the EGI can be difficult to understand. In order to address these problems, we propose a new form of Gaussian Image showing both views which is highly interactive. The views are linked and annotated to clarify correspondences between them. We call this new form an *Interactive Gaussian Image (IGI)*.

### 5.1 Color Mapping

When viewing the two views it is difficult to determine where a point in one view is on the other view. To make this correspondence clearer, we redundantly map the normal to a color. This mapping could logically be on the normals in either object space or camera space. We allow the user to choose which will be used. If the object space normals are chosen, a color can be associated with each vertex based on the normal. Two possible color scales spring to mind for this task. The most intuitive color scale maps the angle of the normal to hue, and the saturation to the remaining direction. Another approach is to vary red, green and blue based on the value of the x, y and z coordinates respectfully. We implemented both

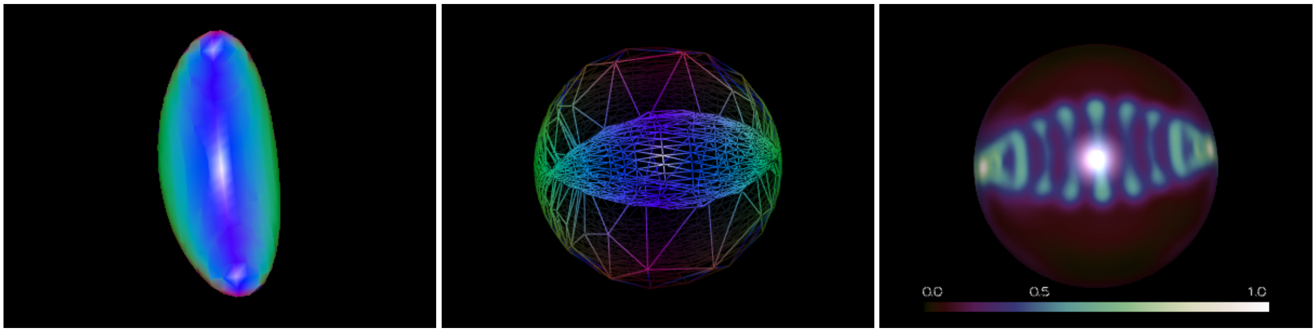


Figure 2: The model-view (left) and two different Gauss-views (center – mesh view and the splatted Gauss map – right) of an implicitly created shape. This viewpoint is a frontal perspective of the saddle-shaped section of a simple bean shape. The highlight on the fold of the meshed Gauss map indicates that there are three separate locations on the model that share the same normal, showing three highlights (left).

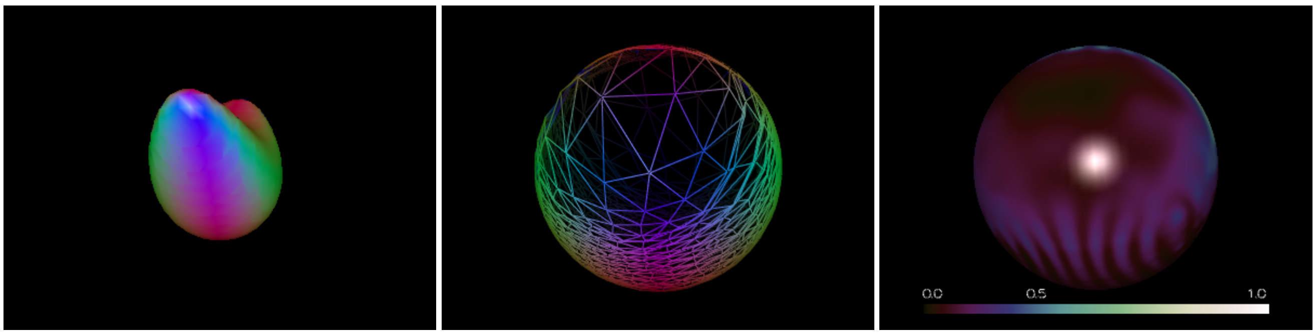


Figure 3: A different perspective on the test shape. Single coverage of the Gauss map at the highlight (right) suggests only one specular highlight on the model view (left). The object is self occluding in this view (left), indicating a tangential view of the Gauss map fold (center).

approaches. Surprisingly, the RGB-based scale color scale seemed to work much better then the hue-saturation color scale. The more intuitive hue-saturation scale suffered from ambiguities in the contribution of colormapping and lighting to the resulting surface color. The RGB-based color scale is used in the figures shown.

Because the normals are the same across the triangles in both views, the specular highlights fall onto the same triangles. During interaction, this specular highlight can be used as a pointer to explore interesting areas of the model and the Gaussian image.

### 5.2 Linked Camera Control

In an orthographic view, half of the Gaussian Sphere is visible. Similarly with a convex object, the part of the surface which has a normal facing toward the viewer is visible. All of the visible points on the surface are mapped to visible points on the Gaussian sphere. As the model rotates, the EGI rotates in the same matter, since parallel surface normals on both the model and the Gaussian sphere, remain parallel during rotation[4]. To achieve these properties we use orthographic views where the camera in the model-view and the Gauss-view are linked so that the same part of the object is being seen in each view. Any rotation which is performed on the camera is carried out on both views, but translations and zooming are carried out on the individual views.

### 5.3 Brushing

When the model is complex and non-convex (for instance the bunny shown in Figure 4), the Gaussian image can still be confusing. To further clarify correspondences between views, we provide a brushing mechanism which allows the user to select parts of the object in one view and see those parts highlighted in the other view. During brushing, selected polygons are highlighted, while other polygons are faded. Brushing enables the user to focus on a local area, while still showing the whole mesh for context.

## 6 DISCUSSION

We are investigating dynamic visualization of the Gauss map. Interactive control plays an important role in the exploratory power of visualization, and we apply it here to the study of solid models. Figures 2 and 3 show still views of these visualization techniques applied to a simple bean shape which has elliptical and hyperbolic regions on its surface. In both examples, the model view is on the left, a mesh representation of the Gauss map is in the center, and the splatted representation of the Gauss map is on the right. Inspecting these images, it is clear that the triangle mesh can clearly be seen in the Gauss map.

Comparing the two views of the same object shows the value of the Gauss map as a visualization of surface geometry. The viewpoint in figure 2 is a frontal perspective of the hyperbolic, saddle shaped, section. The hyperbolic region on the model corresponds to the triply covered, folded region on the Gauss map, with the folds mapping to the parabolic curve separating the elliptical and the hyperbolic surface patches. The specular highlight on the fold of the meshed Gauss map indicates that there are three separate locations on the model with the same normal. Inspecting the model-view on the left, there are indeed three separate specular highlights. In figure 3, the highlight is on a singly covered area of the Gauss map, and the model view correctly shows a single highlight. Specular highlights will split or annihilate along parabolic curves. These effects can be easily explained using this spherical projection of surface normals.

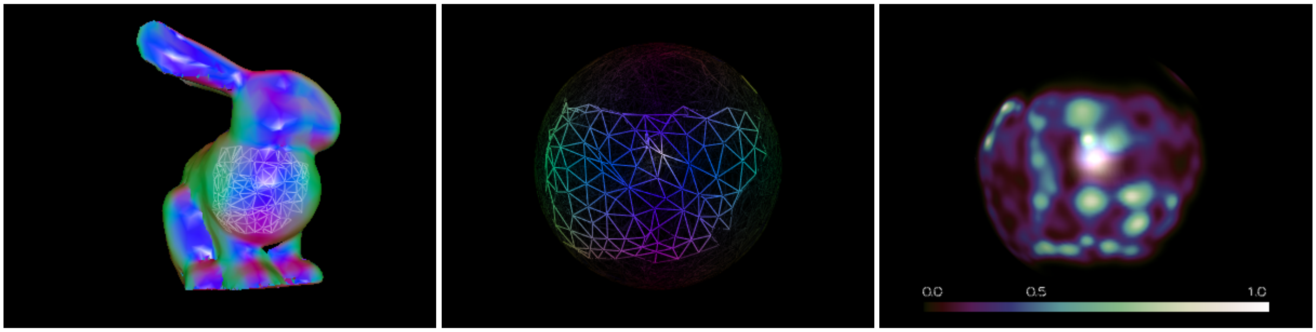


Figure 4: An example of brushing a complex model. When a patch of polygons in the model-view (left) is selected, the corresponding patch is highlighted in the Gauss maps (center and right). This technique helps disambiguate areas, focusing the visualization or regions of interest.

When viewed under dynamic interactive control, the visualization can be striking.

Beyond the behavior of specular and environment-mapped reflections, the Gauss map also reveals surface properties that lead to self occlusion events. Being able to view hyperbolic patches along the normal direction (as in figure 2) and a tangent direction (as in figure 3) and all the intermediate gradations improves the understanding of silhouette management and view-dependent rendering.

Aside from interactive viewpoint control, we had mixed success with other visualization cues. The splatted EGI does provide useful information about the object, though the visualization does not lend itself to immediate comprehension. Brushing of complex models is effective at focusing the visualization, but it is easy to overwhelm the user with detail, suggesting new work on navigation strategies. While the vertex view achieves many of the desired goals, it still needs refinement and experimentation to see if new methods will result in better visualizations. The one unanswered question is how to effectively and clearly deal with areas on the sphere where multiple areas of the model get mapped.

# 7 CONCLUSIONS AND FUTURE WORK

Interactive control of the simultaneous views of the model and its Gauss map create a compelling visualization of the geometric properties of solid shapes. Dynamic visualization of rendering events such as the creation and annihilation of specular highlights, particularly along geometric features such as parabolic curves, lends new understanding to the polygonal models that approximate smooth objects. Our *Interactive Gaussian Image* can be an effective tool for studying shape. The complexity of surface shape can overwhelm our representations. Nevertheless, our visualizations are valuable teaching aids for differential geometry and the exploration of computer graphic models.

We intend both applications and enhancements of these visualizations. Separate interactive control of lighting and viewing direction should permit independent explorations of the visual events that occur through changes in shading and illumination. More importantly, we intend to visualize the process of mesh simplification using tools such as the Gauss map, exploring different algorithms through dynamic views.

# Acknowledgements

This work supported in part by grants from the National Science Foundation (9996043, 0081581, and 0105631) and the Department of Defense (CADIP).

# References

[1] Manfredo P. do Carmo. *Differential Geometry of Curves and Surfaces*. Prentice-Hall, Inc., Upper Saddle River, New Jersey, 1976.

[2] Eitan Grinspun and Peter Schroder. Normal bounds for subdivision-surface interference detection. In *Proceedings of Visualization 2001.*, pages 333–340. IEEE, 2001.

[3] Aaron Hertzmann and Denis Zorin. Illustrating smooth surfaces. In Kurt Akeley, editor, *SIGGRAPH 2002 Conference Proceedings*, Annual Conference Series, pages 517–526. ACM SIGGRAPH, The Association for Computing Machinery, August 2000.

[4] B. K. P. Horn. Extended gaussian images. In Lee S. Baumann, editor, *Image Understanding Workshop (New Orleans, LA, October 3-4, 1984)*, pages 72–89. Defense Advanced Research Projects Agency, Science Applications International Corp., 1984.

[5] Katshuki Ikeuchi, Berthold K. P. Horn, Shigemi Nagata, Tom Callahan, and Oded Feingold. Picking up an object from a pile of objects. pages 139–166, 1983.

[6] Katsushi Ikeuchi and Martial Hebert. Spherical representations: from EGI to SAI. Technical report, CS Dept., Carnegie Mellon U., October 1995. CMU-CS-95-197.

[7] Victoria Interrante, Henry Fuchs, and Stephen M. Pizer. Conveying the 3d shape of smoothly curving transparent surfaces via texture. *IEEE Transactions on Visualization and Computer Graphics*, 2(3), 1997.

[8] Jan J. Koenderink. *Solid Shape*. MIT Press, Cambridge, Massachusetts, 1990.

[9] L.A. Lysternik. *Convex Figures and Polyhedra*. Dover Publications, New York, 1963.

[10] Bui-Tuong Phong. Illumination for computer generated pictures. *CACM June 1975*, 18(6):311–317, 1975.

[11] H.T. Tanaka, M. Ikeda, and H. Chiaki. Automatic face and gesture recognition. In *Pattern Recognition, 2000. Proceedings. 15th International Conference on*, pages 372–377. IEEE, 1998.

[12] Lee Westover. Footprint evaluation for volume rendering. *Computer Graphics*, 24(4):367–376, August 1990.